First-order restoration of SU(N_f)×SU(N_f) chiral symmetry with large N_f and Electroweak phase transition

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It has been argued by Pisarski and Wilczek that finite temperature restoration of the chiral symmetry SU(N_f)×SU(N_f) is first-order for $N_f \geq 3$. This type of chiral symmetry with a large N_f may appear in the Higgs sector if one considers models such as walking technicolor theories. We examine the first-order restoration of the chiral symmetry from the point of view of the electroweak phase transition. The strength of the transition is estimated in SU(2)×U(1) gauged linear sigma model by means of the finite temperature effective potential at one-loop with the ring improvement. Even if the mass of the neutral scalar boson corresponding to the Higgs boson is larger than 114 GeV, the first-order transition can be strong enough for the electroweak baryogenesis, as long as the extra massive scalar bosons (required for the linear realization) are kept heavier than the neutral scalar boson. Explicit symmetry breaking terms reduce the strength of the first-order transition, but the transition can remain strongly first-order even when the masses of pseudo Nambu-Goldstone bosons become as large as the current lower bound of direct search experiments.

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I. INTRODUCTION

The standard model (SM) fulfills all three Sakharov conditions [1] for generating a baryon asymmetry in the Universe [2–4]. However, the model fails to explain the value of the asymmetry, $n_B/s = (8.7 \pm 0.3) \times 10^{-11}$, measured through the Cosmic Microwave Background [5], or the value required for the primordial nucleosynthesis [6], for two reasons. The first reason is that CP violation from the Kobayashi-Maskawa mechanism [7] is highly suppressed [8–13]. The second reason is that the electroweak phase transition (EWPT) is not strongly first-order. The experimental lower bound on the Higgs mass, $m_H > 114$ GeV [14], implies that there is no EWPT in the SM [15–18]. Consequently, sphaleron-induced (B+L)-violating interactions are not sufficiently suppressed in the broken phase and wash out the baryon asymmetry. If the physics at the electroweak scale could explain the baryon asymmetry in the Universe, the better understanding of the structure of the Higgs sector and the source of CP violation would be required.

Various extensions of the Higgs sector have been investigated from the above point of view: these include two Higgs doublet model [19–30], minimal SUSY standard model (MSSM) [31–48], MSSM with an extra singlet scalar [49–57], the SM with a low cut-off [58–60], and so on. The orginal Higgs sector of the SM, if the electroweak interactions is turned off, is nothing but the O(4) linear sigma model and its finite temperature phase transition is second order, which is governed by the Wilson-Fisher IR-stable fixed point. It is the effect of the gauge interaction which makes the fixed point IR-unstable and causes a first-order phase transition [61]. Once the Higgs sector is extended, the number of the scalar fields is increased and there appear additional quartic couplings among them. The fixed points of the multiple quartic coupling constants may be IR-unstable and one can expect a first-order phase transition even in the pure scalar sector [62, 63]. A related approach is to consider the single Higgs doublet model (the O(4) model) with the dimension-six or higher operators [58–60]. The quartic coupling is then assumed to be negative so that the model is out of the domain of the Wilson-Fisher fixed point. Such higher dimensional operators may be induced by the effect of heavy particles coupled to the Higgs doublet, or more generally, by the effect of a certain dynamical system behind the Higgs sector.

In this paper, we consider the finite temperature restoration of SU(N_f)×SU(N_f) chiral symmetry with a large N_f from the point of view of the electroweak phase transition. It has been argued by Pisarski and Wilczek that *if $N_f \geq 3$, the restoration of the chiral symmetry at finite temperature should be first-order*, through the renormalization group analysis of linear sigma model as a low energy effective theory of QCD [64] [63, 65–71]. The chiral symmetry with a large N_f may appear in the Higgs sector as a symmetry of a certain underlying dynamical system such as walking technicolor theories [72–82]. We will examine whether the first-order phase transition associated with the chiral symmetry restoration can be strong enough for the electroweak baryogenesis to operate. A similar consideration has been performed by Appelquist et al. in [70].

Following Pisarski and Wilczek, we consider the renormalizable SU(N_f)×SU(N_f) linear sigma model with a large N_f as a low energy effective theory of a certain un-

derlying dynamical system such as walking technicolor theories. In adopting the model for the Higgs sector, we couple the SU(2)×U(1) electroweak interactions and introduce the explicit symmetry breaking terms which give rise to the masses of the extra Nambu-Goldstone (NG) bosons. We will examine the chiral symmetry restoration in the model by the ϵ-expansion [63–69] and by the finite temperature effective potential obtained at one-loop in the ring-improved perturbation theory [83–92]. The sphaleron energy will be also re-examined within this model in order to clarify the criterion for the strength of the first-order transition [93, 94]. In [70], the finite-temperature effective potential has been examined in a large N_f expansion, but only for the case of the pure linear sigma model without the gauge interactions nor the explicit symmetry breaking terms.

Since the critical behavior of first-order phase transition at finite temperature that we concern, is not universal in general, the result of our analysis would depend on our choice of low energy effective theory, where a certain truncation of fields and operators has to be done. Moreover, for the validity of the perturbation theory at finite temperature, the gauge- and scalar- coupling constants must satisfy certain constraints. It turns out that one cannot push the masses of the scalar fields to so large values as expected in walking technicolor theories: the masses of the scalar fields are up to 300 GeV and in particular the mass of the neutral scalar field which corresponds to the Higgs field is up to 150 GeV. Our analysis, therefore, must be qualitative, just showing a possibility to realize strongly first-order electroweak phase transition required for the electroweak baryogenesis. We leave more thorough quantitative study of this possibility for other non-perturbative methods like the Monte Calro simulation in lattice gauge theory [95–99].

We do not address, in this paper, the question about a possible new source of CP violation which is required for the electroweak baryogenesis. The sector which is responsible for the flavor physics, in particular, the generation of the masses of quarks and leptons, may well affect the dynamics of the chiral symmetry breaking/restoration. This effect of the flavor sector may be partly incorporated into the linear sigma model through the Yukawa couplings to quarks and leptons and the explict symmetry breaking terms which give rise to the masses and the interactions of the pseudo Nambu-Goldstone (NG) bosons. However, the experimental constraints on the flavor sector make it quite non-trivial to build a model of dymanical electroweak symmetry breaking which takes account of the flavor physics. (See [100] for recent progress in constructing a complete theory of fermion masses in the context of extended technicolor theories.) In view of this situation, we simply omit the Yukawa couplings to quarks and leptons (and the CP violating phases) and consider only the effect of the explicit symmetry breaking terms in the scalar fields. We choose the range and the pattern of the mass spectrum of the pseudo NG bosons so that the masses are within the allowed region of the direct

search experiments [101] and the contribution to the S parameter is not positive [102, 103].

At zero temperature, the phase transition associated with the breaking/restoration of $SU(N_f) \times SU(N_f)$ chiral symmetry with a large N_f has been examined by several authors [104–111]. In $SU(N)$ vector-like gauge theories with N_f massless flavors, there appears a non-trivial IR fixed point (the Banks-Zaks fixed point) [104] in the weak coupling perturbation theory for N_f large, but slightly less than $11N/2$. As N_f is lowered, the value of the fixed-point coupling becomes larger. If the Banks-Zaks fixed point persists to larger coupling, it may exceed the critical value for chiral symmetry breaking. It has been argued that this "phase transition in the number of flavors N_f" shows the peculiar nature: the transition is not second order even though the order parameter changes continuously. The critical behavior is not universal, where, from the side of the broken phase, the entire spectrum collapses to zero mass and contributes to any correlation functions. Then, one cannot reduce the theory to an effective low-energy scalar theory. However, in a real life, N_f is supposed to be a fixed integer between the "critical value" $N_f^c = N(100N^2 - 66)/(25N^2 - 15)$ and $11N/2$. Although there would be some very light particles besides the NG bosons, one may assume a description by a certain low energy effective theory which may include these light particles. The linear sigma model we consider in this paper is hopefully one of such low energy effective theories [134].

This paper is organized as follows. In section II, we describe how to introduce the SU(2)×U(1) gauge interactions and the explicit symmetry breaking terms to the $SU(N_f) \times SU(N_f)$ linear sigma model. In section III, we examine the (non-gauged) model by the ϵ-expansion. We will see that the model shows a first-order transition in the parameter region close to the "stability-boundary". Then we proceed to the analysis of the SU(2)×U(1) gauged linear sigma model. In section IV, we examine the sphaleron energy. In section V, we obtain the finite temperature effective potential at one-loop with the ring-improvement and examine it both analytically (in high-temperature expansion) and numerically. Section VI is devoted to a summary and discussions.

II. $\mathbf{SU}(N_f) \times \mathbf{SU}(N_f)$ LINEAR SIGMA MODEL AND THE ELECTROWEAK INTERACTIONS

A. $\mathbf{SU}(N_f) \times \mathbf{SU}(N_f)$ linear sigma model

We consider the renormalizable linear sigma model which is defined by the following lagrangian:

$$\begin{aligned}\mathcal{L}_\Phi &= \mathrm{tr}(\partial_\mu \Phi^\dagger \partial^\mu \Phi) - m_\Phi^2 \, \mathrm{tr}\, \Phi^\dagger \Phi \\ &\quad - \frac{\lambda_1}{2}(\mathrm{tr}\, \Phi^\dagger \Phi)^2 - \frac{\lambda_2}{2}\, \mathrm{tr}(\Phi^\dagger \Phi)^2.\end{aligned} \quad (1)$$

The field $\Phi(x)$ is a $N_f \times N_f$ complex matrix which transforms under the chiral symmetry as

$$\Phi \to e^{i\alpha} g_L \Phi g_R^{-1}, \quad g_L, g_R \in SU(N_f); \ e^{i\alpha} \in U(1)_A. \quad (2)$$

For stability, the quartic couplings should statisfy the following conditions at tree level: $\lambda_2 > 0, \lambda_1 + \lambda_2/N_f > 0$.

We assume that the chiral symmetry breaks down to the diagonal subgroup $SU(N_f)_V$ by the vacuum expectation value (VEV) of $\Phi(x)$:

$$\langle\Phi\rangle = \frac{\phi_0}{\sqrt{2N_f}}\mathbb{1}, \quad (3)$$

where $\mathbb{1}$ is the $N_f \times N_f$ unit matrix. At tree level, the VEV is determined by the effective potential:

$$V_0(\phi) = \frac{1}{2}m_\Phi^2\phi^2 + \frac{1}{8}\left(\lambda_1 + \frac{\lambda_2}{N_f}\right)\phi^4. \quad (4)$$

For $m_\Phi^2 < 0$, it is given by

$$\phi_0 = \sqrt{\frac{-2m_\Phi^2}{\lambda_1 + \lambda_2/N_f}}. \quad (5)$$

Around the VEV, we may parametrize the fluctuation of $\Phi(x)$ as follows:

$$\Phi(x) = \frac{\phi + h + i\eta}{\sqrt{2N_f}}\mathbb{1} + \sum_{\alpha=1}^{N_f^2-1}(\xi^\alpha + i\pi^\alpha)T^\alpha, \quad (6)$$

where T^α ($\alpha = 1, \cdots, N_f^2 - 1$) are the generator of $SU(N_f)$ with the normalization $\text{tr}(T^\alpha T^\beta) = \delta^{\alpha\beta}/2$. The fields $h, \eta, \xi^\alpha, \pi^\alpha$ acquire masses at the tree level as summerized in TABLE I, where, for notational simplicity, we use the following abbreviations:

$$a_h = \frac{3}{2}(\lambda_1 + \lambda_2/N_f), \quad (7)$$

$$a_\xi = \frac{1}{2}(\lambda_1 + 3\lambda_2/N_f), \quad (8)$$

$$a_\eta = a_\pi = \frac{1}{2}(\lambda_1 + \lambda_2/N_f), \quad (9)$$

and

$$b_h = a_h - a_\pi = (\lambda_1 + \lambda_2/N_f), \quad (10)$$

$$b_\xi = a_\xi - a_\pi = (\lambda_2/N_f). \quad (11)$$

h, the singlet of $SU(N_f)_V$, corresponds to the Higgs field. The adjoint π^α are the NG bosons of the breaking of $SU(N_f)\times SU(N_f)$, while the singlet η is the NG boson of the breaking of $U(1)_A$. The adjoint ξ^α are the extra massive degrees of freedom of the linear sigma model, which are required for the linear realization of the chiral symmetry. Three of the NG bosons π^α are eaten by the $SU(2)\times U(1)$ gauge bosons when the electroweak interactions are introduced.

particle	$m_i^2(\phi)$	$m_i^2(\phi_0)$	n_i
h	$m_\Phi^2 + a_h\phi^2$	$b_h\phi_0^2$	1
ξ	$m_\Phi^2 + a_\xi\phi^2$	$b_\xi\phi_0^2$	$N_f^2 - 1$
η	$m_\Phi^2 + a_\eta\phi^2$	0	1
π	$m_\Phi^2 + a_\pi\phi^2$	0	$N_f^2 - 1$

TABLE I: The effective masses and the degrees of freedom of the fields in $SU(N_f)\times SU(N_f)$ linear sigma model.

particle	$m_i^2(\phi)$	$m_i^2(\phi_0)$	n_i
h	$m_\Phi^2 - c' + a_h\phi^2$	$b_h\phi_0^2$	1
ξ	$m_\Phi^2 + c' + a_\xi\phi^2$	$b_\xi\phi_0^2 + 2c'$	$N_f^2 - 1$
η	$m_\Phi^2 + c' + a_\eta\phi^2$	$2c'$	1
π	$m_\Phi^2 - c' + a_\pi\phi^2$	0	$N_f^2 - 1$

TABLE II: The effective masses and the numbers of degrees of freedom in $SU(N_f)\times SU(N_f)$ linear sigma model with the $U(1)_A$ breaking term ($N_f=2$).

For $N_f \leq 4$, we may also add the relevant term,

$$\mathcal{L}'_\Phi = c'(\det\Phi + \det\Phi^\dagger), \quad (12)$$

which breaks the $U(1)_A$ symmetry explicitly, but preserves CP symmetry. This term lifts the mass of η to a nonzero value. In the case of $N_f = 2$, the term is quadratic and the mass spectrum changes as in TABLE II. We note that by sending c' to infinity one can decouple the field components ξ and η. In this case, the original field Φ reduces to an O(4) variable,

$$\Phi(x) = \frac{1}{2}\left[(\phi + h)\mathbb{1} + i\sum_{\alpha=1}^{3}\pi^\alpha\sigma^\alpha\right], \quad (13)$$

and also the quartic couplings reduce to the single one in the combination $\lambda_1 + \lambda_2/2$ due to the identity $\text{tr}(\Phi\Phi^\dagger)^2 = (\text{tr}\Phi\Phi^\dagger)^2/2$. Thus the model reduces to the O(4) linear sigma model. For $N_f = 3, 4$, the term contributes to the effective potential as follows:

$$V_0(\phi) = \frac{1}{2}m_\Phi^2\phi^2 + \frac{1}{8}\left(\lambda_1 + \frac{\lambda_2}{N_f}\right)\phi^4 - 2c'\frac{\phi^{N_f}}{(\sqrt{2N_f})^{N_f}}. \quad (14)$$

The addition of the cubic term in the case of $N_f = 3$ may enhance the first-order chiral phase transition.

B. SU(2)×U(1) gauged linear sigma model

Next we consider the coupling of the electroweak interactions to the linear sigma model. $SU(2)\times U(1)$ gauge interactions may be introduced through the minimal coupling to $\Phi(x)$ as

$$D_\mu\Phi = \partial_\mu\Phi - igA_\mu^a R(T_L^a)\Phi - ig'B_\mu\left[R(Y_L)\Phi - \Phi R(Y_R)\right], \quad (15)$$

particle	$m_i^2(\phi)$	$m_i^2(\phi_0)$	n_i
W	$\frac{g^2}{4}\phi^2$	$\frac{g^2}{4}v^2$	6
Z	$\frac{g^2+g'^2}{4}\phi^2$	$\frac{g^2+g'^2}{4}v^2$	3

TABLE III: The mass and the number of degrees of freedom of the gauge bosons in the one-family type of the SU(2)×U(1) gauged linear sigma model.

where $R(T_L^a)$ $(a=1,2,3)$ and $R(Y_L), R(Y_R)$ are the generators of SU(2) and U(1), respectively, represented by 2×2 block-diagonal traceless hermitian matrices (assuming N_f is even). Here we consider two types of representations inspired by the one-family model and the partially-gauged model [82] of technicolor theories:

particle	$m_i^2(\phi)$	$m_i^2(\phi_0)$	n_i
W	$\frac{g^2}{2N_f}\phi^2$	$\frac{g^2}{4}v^2$	6
Z	$\frac{g^2+g'^2}{2N_f}\phi^2$	$\frac{g^2+g'^2}{4}v^2$	3

TABLE IV: The mass and the number of degrees of freedom of the gauge bosons in the partially-gauged type of the SU(2)×U(1) gauged linear sigma model.

(a) one-family type

$$R(T_L^a) = \frac{\sigma^a}{2} \otimes \mathrm{diag}(1,\cdots,1), \qquad (16)$$

$$R(Y_L) = 0, \qquad (17)$$

$$R(Y_R) = \frac{\sigma^3}{2} \otimes \mathrm{diag}(1,\cdots,1). \qquad (18)$$

(b) partially-gauged type

$$R(T_L^a) = \frac{\sigma^a}{2} \otimes \mathrm{diag}(1,0,\cdots), \qquad (19)$$

$$R(Y_L) = 0, \qquad (20)$$

$$R(Y_R) = \frac{\sigma^3}{2} \otimes \mathrm{diag}(1,0,\cdots). \qquad (21)$$

For these two types, the masses of the gauge bosons, $W^\pm$ and Z, are obtained as summerized in TABLE III and TABLE IV, respectively. We note that in these two types, there are different relations of ϕ to the weak scale $v = 246\mathrm{GeV}$. Namely, we have
(a) one-family type

$$\phi_0 = v \qquad \text{[one-family type]}. \qquad (22)$$

(b) partially-gauged type

$$\phi_0 = \sqrt{\frac{N_f}{2}}\, v \quad \text{[partially-gauged type]}. \qquad (23)$$

For the one-family type with $N_f = 8$, one may consider to couple the color gauge interaction. In this case, the minimal coupling Eq. (15) should be modified as follows:

$$\begin{aligned}
D_\mu\Phi = {}& \partial_\mu\Phi - igA_\mu^a R(T_L^a)\Phi \\
& - ig'B_\mu \left[R(Y_L)\Phi - \Phi R(Y_R)\right] \\
& - ig_3 G_\mu^A \left[R(S^A), \Phi\right],
\end{aligned} \qquad (24)$$

where

$$R(T_L^a) = \frac{\sigma^a}{2} \otimes \mathrm{diag}(1,1,1,1), \qquad (25)$$

$$R(Y_L) = \mathbb{1}_2 \otimes \mathrm{diag}(\tfrac{1}{6},\tfrac{1}{6},\tfrac{1}{6},-\tfrac{1}{2}), \qquad (26)$$

$$\begin{aligned}
R(Y_R) = {}& \frac{\sigma^3}{2} \otimes \mathrm{diag}(1,1,1,1) \oplus \\
& \mathbb{1}_2 \otimes \mathrm{diag}(\tfrac{1}{6},\tfrac{1}{6},\tfrac{1}{6},-\tfrac{1}{2}).
\end{aligned} \qquad (27)$$

and $R(S^A)(A=1,\cdots,8)$ are the generators of SU(3)$_c$ represented by the Gell-Mann matrices λ^A as

$$R(S^A) = \mathbb{1}_2 \otimes \begin{pmatrix} \frac{\lambda^A}{2} & \vdots \\ \cdots & 0 \end{pmatrix}. \qquad (28)$$

Accordingly, the field Φ may be decomposed into the irreducible representations of SU(3). In terms of 2×2 complex matrix valued fields, ϕ_S, $\phi_{S'}$, $\phi_O^A(A=1,\cdots,8)$, ϕ_T^i, $\phi_{\bar{T}}^i(i=1,2,3)$, we have the following decomposition,

$$\Phi = \begin{pmatrix} \frac{1}{2}\phi_S\mathbb{1}_3 + \frac{1}{2\sqrt{3}}\phi_{S'}\mathbb{1}_3 + \phi_O^A\frac{\lambda^A}{\sqrt{2}} & \phi_T \\ {}^t\phi_{\bar{T}} & \frac{1}{2}\phi_S - \frac{\sqrt{3}}{2}\phi_{S'} \end{pmatrix}. \qquad (29)$$

where $\mathbb{1}_3 = \mathrm{diag}(1,1,1)$.

C. Explicit symmetry breaking terms

In order to give rise to the masses to the extra $N_f^2 - 4$ NG bosons, we introduce the terms which break the chiral symmetry explicitly. For simplicity, we consider the minimal breaking for the one-family type, which preserves SU(2)$_L$×SU(2)$_R$×SU($N_f/2$)$_V$ subgroup of the original chiral symmetry so that it is consistent with the gauge symmetries SU(2)×U(1)(× SU(3)). For this purpose, we may parametrize the fluctuation of $\Phi(x)$ by 2×2 complex matrix valued fields as follows (assuming N_f is even):

$$\Phi(x) = \phi_S(x)\otimes\sqrt{\frac{2}{N_f}}\,\mathbb{1}_{N_f/2} + \sum_{\alpha=1}^{(N_f/2)^2-1} \phi_P^\alpha(x)\otimes S^\alpha, \qquad (30)$$

and

$$\begin{aligned}
\phi_S(x) &= (\phi + h(x) + i\eta(x))\frac{\mathbb{1}_2}{2} + (\xi_S^a(x) + i\pi_S^a(x))\frac{\sigma^a}{2}, \\
\phi_P^\alpha(x) &= \xi_P^\alpha(x) + i\pi_P^\alpha(x) \quad (\xi_P^{\alpha\dagger} = \xi_P^\alpha, \pi_P^{\alpha\dagger} = \pi_P^\alpha), \quad (31)
\end{aligned}$$

where S^α $(\alpha = 1,\cdots,(N_f/2)^2 - 1)$ are the generator of SU($N_f/2$) with the normalization $\mathrm{tr}(S^\alpha S^\beta) = \delta^{\alpha\beta}$.

particle	$m_i^2(\phi)$	$m_i^2(\phi_0)$	n_i
h	$m_\Phi^2 + \Delta m^2 - c + a_h\phi^2$	$b_h\phi_0{}^2$	1
ξ_S	$m_\Phi^2 + \Delta m^2 + c + a_\xi\phi^2$	$b_\xi\phi_0{}^2 + 2c$	3
η	$m_\Phi^2 + \Delta m^2 + c + a_\eta\phi^2$	$2c$	1
π_S	$m_\Phi^2 + \Delta m^2 - c + a_\pi\phi^2$	0	3
ξ_P	$m_\Phi^2 + a_\xi\phi^2$	$b_\xi\phi_0{}^2 + c - \Delta m^2$	$N_f^2 - 4$
π_P	$m_\Phi^2 + a_\pi\phi^2$	$c - \Delta m^2$	$N_f^2 - 4$

TABLE V: The effective masses and the numbers of degrees of freedom in SU(N_f)×SU(N_f) linear sigma model with the symmetry breaking terms.

$\phi_S(x)$ in the singlet of SU($N_f/2$)$_V$ and $\phi_P^\alpha(x)$ in the adjoint representation of SU($N_f/2$)$_V$ transform under the SU(2)$_L$×SU(2)$_R$ transformation as

$$\phi_S \to U_L\phi_S U_R^{-1}, \ \phi_P^\alpha \to U_L\phi_P^\alpha U_R^{-1}, \qquad (32)$$

where $U_L, U_R \in$ SU(2). Then we consider the term

$$\Delta\mathcal{L}_{sb} = \Delta\mathcal{L}_2 + \Delta\mathcal{L}_4, \qquad (33)$$

where

$$\Delta\mathcal{L}_2 = -\Delta m^2\mathrm{tr}\{\phi_S^\dagger\phi_S\} + c\{\det\phi_S + c.c.\}$$
$$+ \sum_{\alpha=1}^{(N_f/2)^2-1} c_P\{\det\phi_P^\alpha + c.c.\} \qquad (34)$$

and $\Delta\mathcal{L}_4$ is the quartic coupling term which consists of $\mathrm{tr}\{\phi_S^\dagger\phi_S\}$, $\det\phi_S$, $\det\phi_P^\alpha$, $\mathrm{tr}\{\Phi^\dagger\Phi\}$, and so on.

For further simplicity, we restrict ourselves to the parameter region where the last term of $\Delta\mathcal{L}_2$ and $\Delta\mathcal{L}_4$ vanish identically, including possible one-loop corrections, and assume that the vacuum preserves SU($N_f/2$)$_V$. In this case, the coefficient of the quadratic term of the tree level effective potential Eq. (4) shifts: $m_\Phi^2 \to m_\Phi^2 + \Delta m^2 - c$. The VEV is then given at tree level by

$$\phi_0 = \sqrt{\frac{-2(m_\Phi^2 + \Delta m^2 - c)}{\lambda_1 + \lambda_2/N_f}}. \qquad (35)$$

The masses of the fields $h, \eta, \xi_S, \pi_S, \xi_P, \pi_P$ at tree level are summerized in TABLE V. To escape the vacuum instability, we require $c > 0$ and $c - \Delta m^2 > 0$ at tree level. We note that with this mass spectrum of the pseudo NG bosons (at tree level), the total constribution of the pseudo NG bosons to the S parameter is negative [102, 103].

III. FIRST-ORDER RESTORATION OF SU(N_f)×SU(N_f) CHIRAL SYMMETRY WITH A LARGE N_f IN $d = 4 - \epsilon$

In this section, we examine the critical behavior of the chiral symmetry restoration at finite temperature by the ϵ-expansion in the (non-gauged) SU(N_f)×SU(N_f) linear sigma model. We first review the argument of Pisarski and Wilczek [64] by examining the structure of the fixed points of the renormalization group equations for the quartic couplings λ_1 and λ_2. We then examine the effective potential, following Rudnick [65], Iacobson and Amit [63, 66], in order to see that the transition can actually be first-order.

In $d = 4 - \epsilon$ dimensions, the beta functions of the quartic couplings λ_1 and λ_2 at one-loop are obtained as follows:

$$\beta_1 = -\epsilon\lambda_1 + \frac{3}{8\pi^2}\left(\frac{N_f^2+4}{3}\lambda_1^2 + \frac{4N_f}{3}\lambda_1\lambda_2 + \lambda_2^2\right),$$
$$\beta_2 = -\epsilon\lambda_2 + \frac{3}{8\pi^2}\left(2\lambda_1\lambda_2 + \frac{2N_f}{3}\lambda_2^2\right). \qquad (36)$$

There are three kinds of the fixed point $(\lambda_1^*, \lambda_2^*)$:

(a) $(\lambda_1^*, \lambda_2^*) = (0, 0)$.

(b) $(\lambda_1^*, \lambda_2^*) = 8\pi^2\epsilon\left(\frac{1}{N_f^2+4}, 0\right)$.

(c) $(\lambda_1^*, \lambda_2^*) = 8\pi^2\epsilon\left(c, (1 - 6c)\frac{1}{2N_f}\right) \ (N_f \le \sqrt{3})$,
where c is a solution of the equation:
$(N_f^4 - 8N_f^2 + 27)c^2 + (N_f^2 - 9)c + 3/4 = 0$.

The stability of these fixed points can be examined through the stability matrix defined by $\hat{\beta}_{ij} \equiv \partial\beta_i/\partial\lambda_j$ $(i, j = 1, 2)$:

$$\hat{\beta} = \begin{pmatrix} -\epsilon + \frac{[2(N_f^2+4)\lambda_1^*+4N_f\lambda_2^*]}{8\pi^2} & \frac{[4N_f\lambda_1^*+6\lambda_2^*]}{8\pi^2} \\ \frac{[6\lambda_2^*]}{8\pi^2} & -\epsilon + \frac{[6\lambda_1^*+4N_f\lambda_2^*]}{8\pi^2} \end{pmatrix}. \qquad (37)$$

The fixed point (a), the Gaussian fixed point, is IR-unstable. The fixed point (b) is IR-stable if $N_f < \sqrt{2}$. This corresponds to the Wilson-Fisher fixed point with $O(2N_f)$ critical exponent. When $N_f \ge \sqrt{2}$, it becomes IR-unstable in the λ_2 direction. The fixed point (c) exists if $N_f \le \sqrt{3}$ and is IR-unstable if $N_f < \sqrt{2}$. Therefore, if $N_f > \sqrt{3}$ there is no IR-stable fixed points.

The case of $N_f = 2$ requires a special care. As discussed in the previous section, one can add the quadratic term Eq. (12) and take the O(4) limit. Then the quartic couplings reduce to the single coupling $\lambda_1 + \lambda_2/2$ and the unstable direction no more exists. The fixed point (b) is then the IR-stable Wilson-Fisher fixed point with O(4) critical exponent.

From these considerations, one can see that there is no IR-stable fixed points for $N_f \ge 3$. This suggests that the critical behavior of the restoration of chiral symmetry with $N_f \ge 3$ is not second order, although the possibility is not completely excluded.

In order to see that the transition can actually be first-order, we next examine the effective potential. At one-loop, the effective potential of the linear sigma model is
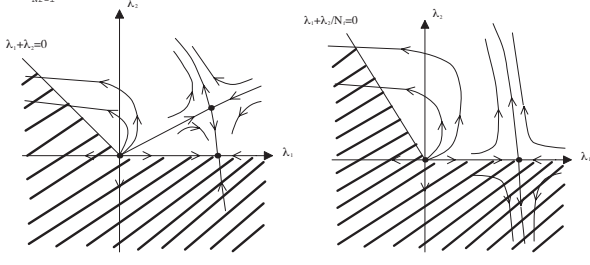
FIG. 1: The flow diagrams in the SU(N_f)×SU(N_f) linear sigma model at $d = 4 - \epsilon$ for $N_f = 1$ (left) and for $N_f > \sqrt{3}$ (right). The arrows indicate the flows of the effective coupling constants toward IR direction. In the shaded region, the effective potential Eq. (4) is unstable.

given by

$$
\begin{aligned}
V_{\text{eff}}(\phi) \;=\; & \frac{1}{2}m_\Phi^2\phi^2 + \frac{1}{8}\left(\lambda_1 + \frac{\lambda_2}{N_f}\right)\phi^4 \\
& + f\left[m_\Phi^2 + \frac{3}{2}\left(\lambda_1 + \frac{\lambda_2}{N_f}\right)\phi^2\right] \\
& + (N_f^2 - 1)f\left[m_\Phi^2 + \frac{1}{2}\left(\lambda_1 + 3\frac{\lambda_2}{N_f}\right)\phi^2\right] \\
& + N_f^2 f\left[m_\Phi^2 + \frac{1}{2}\left(\lambda_1 + \frac{\lambda_2}{N_f}\right)\phi^2\right],
\end{aligned}
\tag{38}
$$

where

$$
f(x) = \frac{1}{64\pi^2}x^2\left(\ln x - \frac{3}{2}\right).
\tag{39}
$$

For the consistency with the ϵ-expansion, it is assumed that

$$
\lambda_1 \sim \lambda_2 \sim \mathcal{O}(\epsilon).
\tag{40}
$$

The first-order phase transition requires that the following conditions are fulfilled at some values of the parameters:

$$
\left.\frac{\partial V_{\text{eff}}}{\partial \phi}\right|_{\phi = \phi_c} = 0, \qquad V_{\text{eff}}(\phi_c) = V_{\text{eff}}(0).
\tag{41}
$$

Now we consider the parameter region on "the stability boundary to $\mathcal{O}(\epsilon)$", following Rudnick [65], Iacobson and Amit [63, 66]:

$$
\lambda_1 + \lambda_2/N_f \sim \mathcal{O}(\epsilon^2).
\tag{42}
$$

There, if one assumes

$$
m_\Phi^2 \sim \mathcal{O}(\epsilon), \qquad \phi_c^2 \sim \mathcal{O}(\epsilon^{-1}),
\tag{43}
$$

the effective potential can be approximated as

$$
\begin{aligned}
V_{\text{eff}}(\phi) \;=\; & \frac{1}{2}m_\Phi^2\phi^2 + \frac{1}{8}\left(\lambda_1 + \frac{\lambda_2}{N_f}\right)\phi^4 \\
& + (N_f^2 - 1)f\left(\frac{\lambda_2}{N_f}\phi^2\right) + \mathcal{O}(\epsilon).
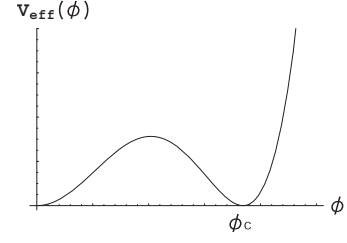\end{aligned}
\tag{44}
$$

FIG. 2: Schematic form of the effective potential at the first-order phase transition.

Then, by denoting

$$
m_\Phi^2 = 2b\epsilon, \qquad \phi_c^2 = c\epsilon^{-1}
\tag{45}
$$

and

$$
\lambda_1 + \frac{\lambda_2}{N_f} = 4\delta\epsilon^2, \qquad \lambda_2/N_f = a\epsilon,
\tag{46}
$$

the conditions Eq. (41) read explicity as

$$
bc + \frac{1}{2}\delta c^2 + \frac{(N_f^2 - 1)}{64\pi^2}(ac)^2[\ln(ac) - 3/2] = 0,
\tag{47}
$$

$$
b + \delta c + \frac{(N_f^2 - 1)}{64\pi^2}(a^2 c)[2\ln(ac) - 2] = 0.
\tag{48}
$$

A solution to these conditions is given by

$$
b \;=\; \frac{(N_f^2 - 1)}{64\pi^2}e^{\left[1/2 - \frac{32\pi^2}{(N_f^2 - 1)}\delta/a^2\right]}a,
\tag{49}
$$

$$
c \;=\; \frac{1}{a}e^{\left[1/2 - \frac{32\pi^2}{(N_f^2 - 1)}\delta/a^2\right]}.
\tag{50}
$$

Thus we can see that the restoration of the chiral symmetry with $N_f \geq 3$ can be first-order indeed in a parameter region close to the stability boundary, Eq. (42).

IV. SPHALERON

In this section, we next examine the sphaleron energy [93, 94] within the SU(2)×U(1) gauged linear sigma model in order to clarify the criterion for the strength of the first-order transition required for the electroweak baryogenesis. For simplicity, we consider the one-family type and omit the U(1) hyper-charge interaction.

When the vacuum preserves SU($N_f/2$)$_V$, we may assume the following anzatz

$$
A_i^a \frac{\sigma^a}{2}dx^i = -\frac{i}{g}f(\xi)dUU^{-1},
\tag{51}
$$

$$
\Phi = \frac{v}{\sqrt{2N_f}}h(\xi)\hat{U},
\tag{52}
$$

$$U = \frac{1}{r} \begin{pmatrix} z & x + iy \\ -x + iy & z \end{pmatrix}, \quad \hat{U} = \begin{pmatrix} U & & & \\ & U & & \\ & & \ddots & \\ & & & U \end{pmatrix},$$

(53)

where $\xi = gvr$ is the dimensionless radial coordinate, $f(\xi)$ and $h(\xi)$ are the unknown functions which subject to the boundary conditions $f(0) = h(0) = 0$ and $f(\infty) = h(\infty) = 1$. If we introduce a 2×2 matrix variable M as

$$M \equiv \frac{v}{\sqrt{2}} h(\xi) U,$$

(54)

the energy functional of our model is written as

$$
\begin{aligned}
E = & \int d^3x \left\{ \frac{1}{4} F_{ij}^a F_{ij}^a + \frac{1}{2} \text{tr}[(D_i M)^\dagger D_i M] \right. \\
& - \frac{1}{4} \left(\lambda_1 + \frac{\lambda_2}{N_f} \right) v^2 \, \text{tr}(M^\dagger M) \\
& \left. + \frac{1}{8} \left(\lambda_1 + \frac{\lambda_2}{N_f} \right) (\text{tr}\, M^\dagger M)^2 \right\},
\end{aligned}
$$

(55)

where v is the VEV determined through the effective potential at tree-level. We note that this result holds true even when one includes the explicit symmetry breaking terms as long as the last term of $\Delta\mathcal{L}_2$ and $\Delta\mathcal{L}_4$ vanishes identically, because the only effect of the symmetry breaking terms is then to shift the quadratic term of the effective potential.

We now compare this result to the energy functional in the SM [93, 94]:

$$
\begin{aligned}
E^{SM} = & \int d^3x \left\{ \frac{1}{4} F_{ij}^a F_{ij}^a + \frac{1}{2} \text{tr}[(D_i M)^\dagger D_i M] \right. \\
& \left. - \frac{1}{2} \lambda v^2 \, \text{tr}(M^\dagger M) + \frac{1}{4} \lambda [\text{tr}(M^\dagger M)]^2 \right\}.
\end{aligned}
$$

(56)

One can easily see that these energy functionals are equal to each other if one identifies the quartic couplings by the relation

$$\frac{1}{2} \left(\lambda_1 + \frac{\lambda_2}{N_f} \right) \leftrightarrow \lambda.$$

(57)

From this fact, one can easily estimate the sphaleron energy in our model. Namely,

$$
\begin{aligned}
E_{sph} &= E_{sph}^{SM} \\
&= \left. \frac{4\pi v}{g} B\left(\frac{\lambda}{g^2} \right) \right|_{\lambda = (\lambda_1 + \lambda_2/N_f)/2},
\end{aligned}
$$

(58)

where $B\left(\lambda/g^2\right)$ is the function given in [93, 94], whose numerical value is of order unity for all range of the variable λ/g^2. Then the condition for the baryon asymmetry not to be washed out is given by

$$\frac{v_c}{T_c} \gtrsim 1,$$

(59)

just same as in the SM. We assume this criterion in the following analysis.

V. ANALYSIS OF FINITE TEMPERATURE EFFECTIVE POTENTIAL

In this section, we examine the critical behavior of the electroweak symmetry restoration at high temperature in the SU(2)×U(1) gauged linear sigma model using the finite temperature effective potential. To evaluate the effective potential, we adopt the ring-improved perturbation theory at one-loop. We first give some analytical results in the high temperature expansion. We next examine the effective potential numerically and estimate semi-quantitatively the strength of the first-order phase transition.

A. Finite temperature effective potential of SU(2)×U(1) gauged linear sigma model

In the one-loop approximation with the ring-improvement, the effective potential of the SU(2)×U(1) gauged linear sigma model may be written as

$$
\begin{aligned}
V(\phi) = & \ V_0(\phi) + V_1^{(0)}(\phi) + \\
& V_1^{(T)}(\phi, T) + V_{\text{ring}}(\phi, T),
\end{aligned}
$$

(60)

where V_0 is the tree-level effective potential, $V_1^{(0)}$ and $V_1^{(T)}$ are the one-loop contributions at zero and finite temperature, respectively, and $V_{\text{ring}}(\phi, T)$ is the contribution from the ring diagrams. V_0, the tree-level effective potential, is given by Eq. (4). When one includes the explicit symmetry breaking terms, Eq. (33), the coefficient of the quadratic term should be shifted as $m_\Phi^2 \to m_\Phi^2 + \Delta m^2 - c$.

$V_1^{(0)}$, the one-loop contribution at zero temperature, is given by

$$V_1^{(0)}(\phi) = \frac{1}{64\pi^2} \sum_i n_i m_i^4(\phi) \left[\ln \frac{m_i^2(\phi)}{\mu^2} - C_i \right],$$

(61)

where i runs over all scalar and vector bosons. n_i and $m_i(\phi)$ are the number of degrees of freedom and the effective masses depending on ϕ, respectively, which are given in TABLE I–V. C_i are the constants given by $C_i = 3/2$ for scalar bosons and $C_i = 5/6$ for gauge bosons. We have chosen the Landau gauge and have used the $\overline{\text{MS}}$ scheme to renormalize the ultraviolet divergences at the renormalization scale μ. At one-loop, the VEV is determined by

$$\left. \frac{\partial[V_0(\phi) + V_1^{(0)}(\phi)]}{\partial\phi} \right|_{\phi = \phi_0} = 0.$$

(62)

At one-loop, the mass of the neutral scalar field, m_h, acquires a rather large correction, the size of which depends on N_f and λ_2/N_f as well as $\lambda_1 + \lambda_2/N_f$. Then, we adopt the following definition for the (renormalized)
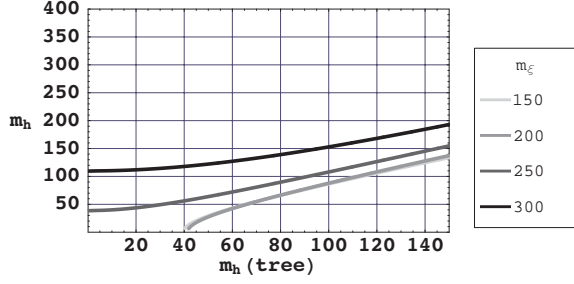
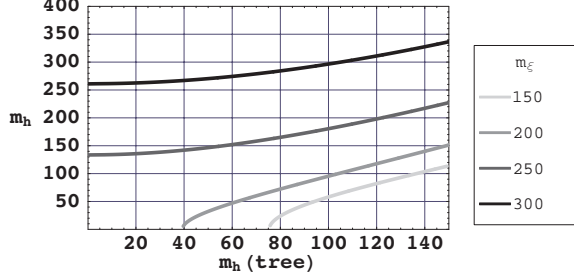FIG. 3: m_h as a function of $(\lambda_1 + \lambda_2/N_f)^{1/2}\phi_0$ for the case $N_f = 4$.

FIG. 4: m_h as a function of $(\lambda_1 + \lambda_2/N_f)^{1/2}\phi_0$ for the case $N_f = 8$.

mass of the neutral scalar field, m_h:

$$m_h^2 \equiv \left. \frac{\partial^2[V_0(\phi) + V_1^{(0)}(\phi)]}{\partial\phi^2} \right|_{\phi=\phi_0}. \qquad (63)$$

As for the mass of the extra scalar fields, m_ξ, we adopt the formula at the tree level:

$$m_\xi^2 \equiv \frac{\lambda_2}{N_f} \phi_0^2. \qquad (64)$$

FIG. 3 shows the plot of m_h as a function of $[m_h]_{\text{tree}} = (\lambda_1 + \lambda_2/N_f)^{1/2}\phi_0$ for several values of m_ξ and $N_f = 4$. FIG. 4 shows a similar plot for $N_f = 8$.

The one-loop contribution at finite temperature, $V_1^{(T)}$, is given by

$$V_1^{(T)}(\phi, T) = \frac{T^4}{2\pi^2} \sum_i n_i J_B[m_i^2(\phi)/T^2], \qquad (65)$$

where i runs over all scalar and vector bosons. J_B is defined by

$$J_B(a) = \int_0^\infty dx \; x^2 \ln\left(1 - e^{-\sqrt{x^2+a}}\right). \qquad (66)$$

In the high temperature limit where $m(\phi) \ll T$, it can be expanded as follows:

$$J_B(m^2/T^2) = -\frac{\pi^4}{45} + \frac{\pi^2}{12}\frac{m^2}{T^2} - \frac{\pi}{6}\left(\frac{m^2}{T^2}\right)^{3/2}$$
$$- \frac{1}{32}\frac{m^4}{T^4}\ln\frac{m^2}{a_b T^2} + \mathcal{O}\left(\frac{m^6}{T^6}\right), \qquad (67)$$

where $a_b = 16\pi^2\exp(3/2 - 2\gamma_E)(\ln a_b \approx 5.4076)$.

In the ordinary perturbation theory at finite temperature, the perturbative expansion breaks down near the critical temperature due to the exitence of the higher-loop IR divergent diagrams in the massless limit. This problem can be partially avoided by resumming the contributions from the ring diagrams which are the most dominant IR contributions at each order of the perturbative expansion, though the coupling constants must satisfy certain constraints to suppress the other higher-loop contributions [85–92].

One can include the contribution of ring diagrams, $V_{\text{ring}}(\phi, T)$, by replacing $m_i^2(\phi)$ in $V_1^{(0)}$ and $V_1^{(T)}$ with the effective T-dependent masses $\mathcal{M}_i^2(\phi, T) \equiv m_i^2(\phi) + \Pi_i$, where Π_i is the self-energy of a particle i in the IR limit where the Matsubara frequency and the momentum of the external fields go to zero and in the leading order of $m_i(\phi)/T$. For the scalar bosons, there are the contributions from the diagrams shown in FIG. 5. For the longitudinal components of the gauge bosons, there are the contributions from the diagram shown in FIG. 6, while the transverse components do not receive the correction. The explicit expressions of the effective T-dependent masses depend on how the gauge interactions are introduced in the sigma model and are given in appendix.

After all, the one-loop ring-improved effective potential is given by

$$V(\phi) = V_0(\phi) + V_1^{(0)}(\phi) + V_1^{(T)}(\phi, T) + V_{\text{ring}}(\phi, T)$$
$$= V_0(\phi) + \sum_i n_i \left[\frac{1}{64\pi^2}\mathcal{M}_i^4(\phi, T)\left\{\ln\frac{\mathcal{M}_i^2(\phi, T)}{\mu^2} - C_i\right\} + \frac{T^4}{2\pi^2}J_B[\mathcal{M}_i^2(\phi, T)\beta^2]\right]. \qquad (68)$$

A comment on the validity of the ring-improved perturbation theory is in order. For simplicity, let us consider
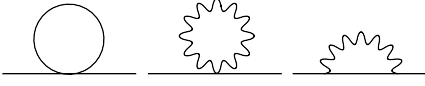
FIG. 5: The one-loop self energy diagrams of scalar bosons. First one comes from the self-interactoin, second and third one are contributions from the SU(2)×U(1) gauge bosons. In the one-family type with $N_f = 8$, there are also the contributions from the gluon.
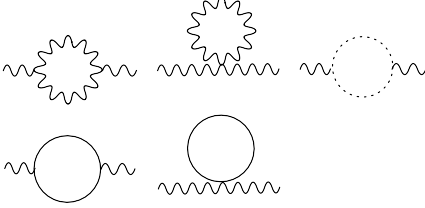

FIG. 6: The one-loop vacuum polarization diagrams of SU(2)×U(1) gauge bosons. First and second one comes from the self interactions, third one is the contribution from the ghost, fourth and fifth one are the contribution from the scalar bosons.

only the contribution of the scalar fields, neglecting the contribution from the gauge fields. This approximation is expected to be valid as long as the scalar fields are heavier than the gauge fields. By inspecting the higher order diagrams for the scalar field self-energies, one can see that the non-ring diagrams are suppressed with respect to the ring diagrams at least by the following factors in the symmetric phase,

$$\beta_{\lambda_1 + \lambda_2/N_f} \equiv \frac{1}{4\pi} \left(\lambda_1 + \frac{\lambda_2}{N_f} \right) \frac{T}{m_{\text{eff}}},$$
$$\beta_{\lambda_2} \equiv \frac{N_f^2}{4\pi} \left(\frac{\lambda_2}{N_f} \right) \frac{T}{m_{\text{eff}}}, \qquad (69)$$

where

$$m_{\text{eff}}^2 \equiv m_\Phi^2 + \frac{T^2}{12} \sum_i n_i a_i = m_\Phi^2 + \frac{b}{12} T^2. \qquad (70)$$

On the other hand, in the broken phase, the suppression factors depend on various mass parameters. Among them, the largest factors in the large N_f limit are given by the loops of the neutral scalar boson h and the NG bosons π and η,

$$\bar{\beta}_{\lambda_1 + \lambda_2/N_f} \equiv \frac{N_f^2}{4\pi} \left(\lambda_1 + \frac{\lambda_2}{N_f} \right) \frac{T}{(m_{\text{eff}}^2 + a_h \phi^2)^{1/2}},$$
$$\bar{\beta}_{\lambda_2} \equiv \frac{N_f^2}{4\pi} \left(\frac{\lambda_2}{N_f} \right) \frac{T}{(m_{\text{eff}}^2 + a_\pi \phi^2)^{1/2}}. \qquad (71)$$

Therefore, in order to guarantee the validity of the ring-improved perturbation theory, it is required that $\beta_{\lambda_1 + \lambda_2/N_f}, \bar{\beta}_{\lambda_1 + \lambda_2/N_f} \ll 1$ and $\beta_{\lambda_2}, \bar{\beta}_{\lambda_2} \ll 1$, while the perturbative expansion at zero temperature is valid for

$N_f^2(\lambda_1 + \lambda_2/N_f)/(4\pi)^2 \ll 1$ and $N_f\lambda_2/(4\pi)^2 \ll 1$. In the following analysis of the effective potential, we will examine whether these conditions are satisfied near the critical temperature.

B. Analytical results in the high temperature expansion

We first examine the effective potential analytically using the high temperature expansion in order to get some insight how ϕ_c/T_c depends on the number of flavor N_f and the quartic couplings $\lambda_1 + \lambda_2/N_f$, λ_2/N_f. For simplicity, we consider only the contribution of the scalar fields as discussed before. In the high temperature limit, the one-loop effective potential can be expanded as

$$V(\phi, T) \simeq \frac{1}{2} m_{\text{eff}}^2 \phi^2 - \frac{T}{12\pi} \sum_i n_i \left[(\mathcal{M}_i^2)^{3/2} - (m_{\text{eff}}^2)^{3/2} \right]$$
$$+ \frac{1}{8} \left(\lambda_1 + \frac{\lambda_2}{N_f} \right) \phi^4 + \mathcal{O}(\mathcal{M}^6/T^6), \qquad (72)$$

where $\mathcal{M}_i^2 = m_{\text{eff}}^2 + a_i \phi^2$. We have subtracted the terms which do not depend on ϕ,

$$V(0, T) = \sum_i n_i \left\{ -\frac{\pi^2 T^4}{90} + \frac{T^2}{24} m_{\text{eff}}^2 - \frac{T}{12\pi} (m_{\text{eff}}^2)^{3/2} \right\}, \qquad (73)$$

and have neglected the term $-\frac{\sum_i n_i}{64\pi^2} \mathcal{M}_i^4 \ln(a_b T^2/\mu^2)$ which may be regarded to be small compared to the tree-level terms. In order that the effective potential is real for all values of ϕ, it must be satisfied that $m_{\text{eff}}^2 = m_\Phi^2 + \frac{b}{12} T^2 \geq 0$, namely,

$$T^2 \geq -12 m_\Phi^2/b \equiv T_1^2. \qquad (74)$$

(At the tree level, $T_1^2 = 6(\lambda_1 + \lambda_2/N_f)\phi_0^2/b$.)

At the temperature T_1, m_{eff}^2 is equal to zero and $\mathcal{M}_i^2 = a_i \phi^2$. Then, it follows

$$V(\phi, T_1) \simeq -\frac{T_1}{12\pi} \sum_i n_i a_i^{3/2} \phi^3 + \frac{1}{8} \left(\lambda_1 + \frac{\lambda_2}{N_f} \right) \phi^4. \qquad (75)$$

At this temperature, the origin of the effective potential is unstable because the ϕ^3 term is negative and there is a minimum point of the potential where the scalar field has a nonzero VEV,

$$\phi_1 = \frac{T_1}{2\pi} \frac{\sum_i n_i a_i^{3/2}}{\lambda_1 + \lambda_2/N_f}. \qquad (76)$$

Above the temperature T_1, m_{eff}^2 becomes positive, and the minimum point appears at the origin other than ϕ_1. This signals the first-order phase transition, and one may expect a first-order phase transition at the temperature $T_c (> T_1)$ where the effective potential satisfies the conditions,

$$\frac{\partial V}{\partial \phi}(\phi_c, T_c) = 0, \qquad V(\phi_c, T_c) = V(0, T_c). \qquad (77)$$

One may consider the ratio ϕ_1/T_1 as an estimate of the strength of the first-order transition and Eq. (76) suggests that the first-order transition becomes stronger as one approaches the stability boundary, $(\lambda_1 + \lambda_2/N_f) \simeq 0$. For a large N_f and $0 \lesssim (\lambda_1 + \lambda_2/N_f) \ll \lambda_2/N_f \lesssim 16\pi^2/N_f^2$, one obtains

$$\frac{\phi_1}{T_1} \simeq 2N_f \left(\frac{\lambda_2/N_f}{\lambda_1 + \lambda_2/N_f}\right) \left[\frac{N_f^2}{16\pi^2}\frac{\lambda_2}{N_f}\right]^{1/2}$$

$$= 2N_f \left[\frac{m_\xi^2}{m_h^2}\right]_{\text{tree}} \left[\frac{N_f^2}{16\pi^2}\frac{\lambda_2}{N_f}\right]^{1/2}, \qquad (78)$$

where we have used the mass relations at the tree level, $m_h^2 = (\lambda_1 + \lambda_2/N_f)\phi_0^2$ and $m_\xi^2 = (\lambda_2/N_f)\phi_0^2$. From Eq. (78), we can see that ϕ_1/T_1 is enhanced for a large N_f and for a large mass ratio m_ξ^2/m_h^2. We note in this case that the ring-improved perturbation theory is valid because one has

$$\bar{\beta}_{\lambda_1+\lambda_2/N_f} \simeq \left(\frac{\lambda_1 + \lambda_2/N_f}{\lambda_2/N_f}\right)^{3/2} \ll 1,$$

$$\bar{\beta}_{\lambda_2} \simeq \left(\frac{\lambda_1 + \lambda_2/N_f}{\lambda_2/N_f}\right)^{1/2} \ll 1. \qquad (79)$$

Thus, the parameter region close to the stability boundary can be accessible within the ring-improved perturbation theory. On the other hand, the high temperature expansion is valid as long as

$$\frac{\phi_1}{T_1} \lesssim \frac{1}{a_\xi^{1/2}} = \left(\frac{1}{\lambda_2/N_f}\right)^{1/2} = \frac{v}{m_\xi}. \qquad (80)$$

Therefore, in the high temperature expansion, the strong first-order transition around the stability boundary can be examined only when m_ξ is small compared to the weak scale v ($= \phi_0$ for the one-family type).

Another interesting parameter region one may consider is the region where the first-order transition turns into a second order transition or a crossover behavior. Such region can be characterized by the condition: $\phi_c/T_c \to 0$. In order to locate such parameter region, we expand the effective potential in terms of ϕ^2/T^2 as

$$V(\phi, T) = T^4 \left[c_1 \frac{\phi^2}{T^2} + c_2 \frac{\phi^4}{T^4} + c_3 \frac{\phi^6}{T^6} + \mathcal{O}\left(\frac{\phi^8}{T^8}\right)\right], \qquad (81)$$

where the coefficients are given within the high temperature expansion by

$$c_1 = \frac{1}{2}\frac{m_{\text{eff}}^2}{T^2}\left(1 - \frac{1}{4\pi}\sum_i n_i a_i \frac{T}{m_{\text{eff}}}\right), \qquad (82)$$

$$c_2 = \frac{1}{8}\left(\lambda_1 + \frac{\lambda_2}{N_f} - \frac{1}{4\pi}\sum_i n_i a_i^2 \frac{T}{m_{\text{eff}}}\right), \qquad (83)$$

$$c_3 = \frac{1}{192\pi}\sum_i n_i a_i^3 \left(\frac{T}{m_{\text{eff}}}\right)^3. \qquad (84)$$

Then Eq. (77) is solved by

$$\frac{\phi_c^2}{T_c^2} = \sqrt{\frac{c_1}{c_3}} = \frac{-c_2}{2c_3} \qquad (85)$$

for $c_1 \geq 0$, $c_2 \leq 0$ and $c_3 > 0$ and the condition $\phi_c/T_c \to 0$ implies that $c_1 = c_2 = 0$. This gives the critical end line:

$$\lambda_1 + \frac{\lambda_2}{N_f} = \left|2 + \frac{2}{N_f^2 - 2}\right|^{\frac{1}{2}} \frac{\lambda_2}{N_f}, \qquad (86)$$

or

$$\left[\frac{m_h^2}{m_\xi^2}\right]_{\text{tree}} = \left|2 + \frac{2}{N_f^2 - 2}\right|^{\frac{1}{2}}. \qquad (87)$$

In this case, $c_1 = 0$ implies that $m_{\text{eff}}/T = \sum_i n_i a_i/4\pi$. Then, we note that

$$\beta_{\lambda_1+\lambda_2/N_f} \simeq \frac{1}{N_f^2}\frac{(\lambda_1 + \lambda_2/N_f)}{(\lambda_1 + \lambda_2/N_f) + (\lambda_2/N_f)}, \qquad (88)$$

$$\beta_{\lambda_2} \simeq \frac{(\lambda_2/N_f)}{(\lambda_1 + \lambda_2/N_f) + (\lambda_2/N_f)}. \qquad (89)$$

Therefore, the ring improved perturbation theory is valid for $(\lambda_1 + \lambda_2/N_f) \gtrsim \lambda_2/N_f (> 0)$. On the other hand, the high temperature expansion is valid for

$$\frac{N_f^2}{4\pi}\left[(\lambda_1 + \lambda_2/N_f) + (\lambda_2/N_f)\right] \lesssim 1. \qquad (90)$$

From Eq. (87), we can see that when the mass of the neutral scalar field becomes large and exceeds those of the extra scalar fields the critical behavior becomes the second order transition, or a crossover behavior. (Within the above anaysis, if the mass ratio $[m_h^2/m_\xi^2]_{\text{tree}}$ becomes larger than the critical value, the transition is second order because c_2 is always positive when $c_1 = 0$.)

These analytical results obtained within the high temperature expansion suggest that as long as $m_h \ll m_\xi (< v)$, one would not encounter the critical end line and the first-order transition can remain strong. Then, the question is how large m_ξ and m_h can be, while keeping the strongly first-order transition with $\phi_c/T_c \simeq \mathcal{O}(1)$. To examine this point, one should go beyond the high temperature expansion.

C. Numerical results

We next examine the effective potential numerically. The condition Eq. (77) is solved for various parameters, m_h, m_ξ and N_f (see Eqs. (63), (64) for definition), and ϕ_c/T_c is evaluated in order to estimate the strength of the first-order phase transition. The renormalization scale is set to the electroweak scale as $\mu = v$. The mass parameter m_Φ^2 is chosen so that at zero temperature the electroweak symmetry is spontaneously broken at $\phi_0 = v$ for the one-family type and at $\phi_0 = \sqrt{(N_f/2)}\, v$ for the partially gauged type.
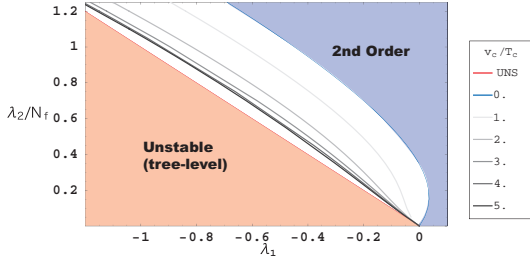
FIG. 7: (Color on line) λ_1–λ_2/N_f diagram of the critical behavior of the chiral symmetry restoration in (non-gauged) SU(N_f)×SU(N_f) linear sigma model with $N_f = 4$. The gray lines show the contours of ϕ_c/T_c. Above $\phi_c/T_c \simeq 0$ (the blue line), the phase transition becomes a second order.
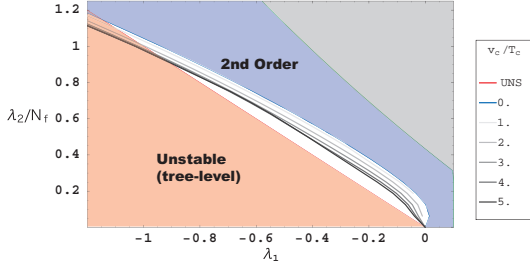

FIG. 8: (Color on line) λ_1–λ_2/N_f diagram of the critical behavior of the chiral symmetry restoration in (non-gauged) SU(N_f)×SU(N_f) linear sigma model with $N_f = 8$.

1. SU(N_f)×SU(N_f) linear sigma model

We first show the numerical result for the non-gauged SU(N_f)×SU(N_f) linear sigma model in the one-family type ($\phi_0 = v$).

FIG. 7 is a diagram which shows the coupling dependence of the critical behavior of chiral symmetry restoration on the λ_1–λ_2/N_f plane for $N_f = 4$. We clearly see that the region of the strongly first-order transition lies above the stability boundary $\lambda_1 + \lambda_2/N_f = 0$, where the contours of ϕ_c/T_c run in almost parallel with the boundary for a large λ_2/N_f, $\lambda_2/N_f \gtrsim 0.7$. The critical end line is located around $\lambda_1 + \lambda_2/N_f \simeq 0.5$ and the transition becomes second order when $\lambda_1 + \lambda_2/N_f$ gets large further. FIG. 8 is a similar diagram for $N_f = 8$.

In FIG. 9, ϕ_c/T_c is plotted as a function of m_h for several values of m_ξ and for $N_f = 4$. The unit of mass is in GeV. We can see that with m_ξ fixed, ϕ_c/T_c decreases and finally vanishes identically as m_h increases. On the other hand, we note that ϕ_c/T_c can remain $\mathcal{O}(1)$ even when m_h increases, if at the same time m_ξ increases keeping the relation $m_h < m_\xi$. In FIG. 10, a similar plot is shown for $N_f = 8$.

In FIG. 11, ϕ_c/T_c is plotted as a function of m_h for several values of N_f with $m_\xi = 250$GeV fixed. For a relatively small m_h, $m_h \lesssim 130$GeV, where ϕ_c/T_c is rather
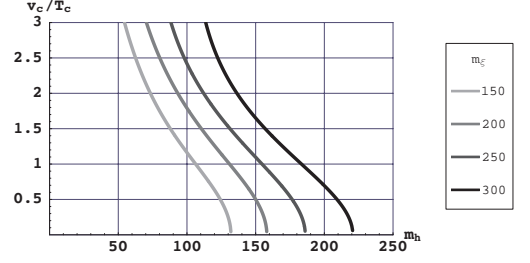

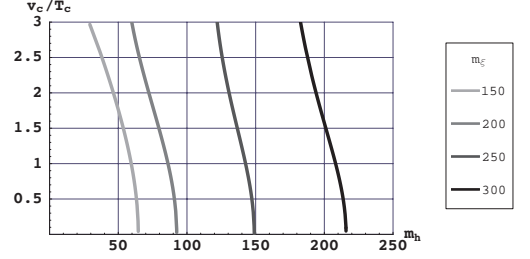FIG. 9: ϕ_c/T_c as a function of m_h for $N_f = 4$.

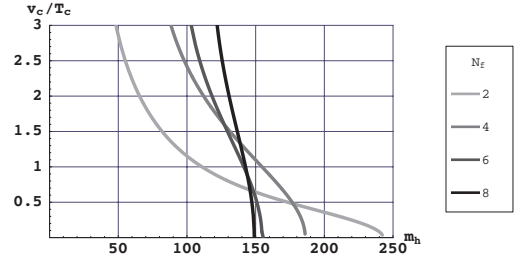FIG. 10: ϕ_c/T_c as a function of m_h for $N_f = 8$.

FIG. 11: N_f dependence of ϕ_c/T_c with $m_\xi = 250$ GeV fixed.

large, we can see a clear dependence on N_f: as N_f increases, the value of m_h which gives the same value of ϕ_c/T_c increases monotonically. However, for a relatively large m_h, where ϕ_c/T_c gets rather small, the dependence on N_f is the opposite. This is because, as one can see from FIG. 8, the critical end line shifts towards the stability boundary and the region of the first-order transition becomes narrower as N_f increases. As the total result, the value of m_h which gives $\phi_c/T_c \simeq \mathcal{O}(1)$ does not depend on N_f so much.

Throughout the above numerical evaluations of ϕ_c/T_c, we observed that $T_c/\mathcal{M}_i(\phi_c, T_c) \lesssim \mathcal{O}(1)$ for $i = h, \pi$, which implies

$$\beta_{\lambda_1 + \lambda_2/N_f} \simeq \frac{N_f^2}{4\pi}\left(\lambda_1 + \frac{\lambda_2}{N_f}\right),$$

$$\beta_{\lambda_2} \simeq \frac{N_f^2}{4\pi}\left(\frac{\lambda_2}{N_f}\right). \tag{91}$$

Then, the ring improved perturbation theory is valid for $\lambda_1 + \lambda_2/N_f \lesssim 4\pi/N_f^2$, and, $\lambda_2/N_f \lesssim 4\pi/N_f^2$. The

second condition can be written in the condition of m_ξ as, $m_\xi/v \lesssim \sqrt{4\pi/N_f^2}$ for the one-family type ($m_\xi/v \lesssim \sqrt{2\pi/N_f}$ for the partially gauged type). Therefore, one cannot push m_ξ to so large values compared to the weak scale v for a reliable evaluation of ϕ_c/T_c within the ring improved perturbation theory. In this paper, we restrict ourselves in the range $m_\xi \lesssim 300$ GeV for both $N_f = 4, 8$. This may not be safe and the approximation may be rather crude for $N_f = 8$ in particular. However, this case is also useful for a comparison and to see the effects of the gauge interactions and the symmetry breaking terms.

2. SU(2)×U(1) gauged linear sigma model

We next show the numerical result for the SU(2)×U(1) gauged linear sigma model.

FIG. 12 is a contour plot of v_c/T_c on the m_h–m_ξ plane for the one-family model with $N_f = 8$. For a comparison, a similar plot for the non-gauged model with $N_f = 8$ is shown in FIG. 13. We can see that the effect of the gauge interactions slightly reduces the value of m_h which gives the same value of v_c/T_c for a fixed m_ξ.

FIG. 14 is a similar contour plot of v_c/T_c for the partially-gauged model with $N_f = 8$. We note that the value of m_h which gives the same value of v_c/T_c for a fixed m_ξ becomes larger in the partially-gauged model. This is mainly due to the difference of the relation between ϕ_0 and v in this model. (See Eq. (23).) The partially gauged model is favorable than the one-family model in realizing a stronger first-order electroweak phase transition.

3. Explicit symmetry breaking terms

Finally, we show the numerical result for the (non-gauged) linear sigma model with the symmetry breaking terms in the case of $N_f = 8$. We omit the effect of the gauge interactions because its effect turns out to be relatively small compared to the effect of the symmetry breaking terms.

FIG. 15 and FIG. 16 show the contours of $\phi_c/T_c = 1$ on the m_h–$m_{\xi s}$ plane for several values of the mass parameters, m_{π_P}, m_η, of the pseudo NG bosons defined by

$$m_\eta^2 = 2c, \qquad m_{\pi_P}^2 = c - \Delta m^2. \tag{92}$$

We choose the range of the masses as $m_\eta, m_{\pi_P} \gtrsim 110$GeV so that it is within the typically allowed region of the direct search experiments [101]. (For a specific Extended Technicolor model, the lower bound can be more severe. Taking into account such cases, we searched a larger parameter region.) We can see that the effect of the symmetry breaking terms is sizable and reduces the strength of the first-order transition for fixed m_h and $m_{\xi s}$. However, the region of the strongly first-order transition still remains for the allowed values of the masses of the pseudo
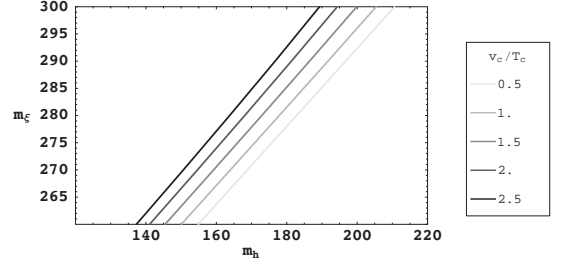

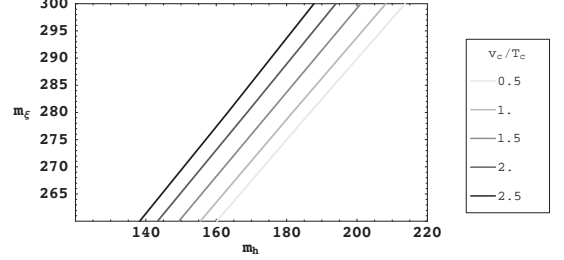FIG. 12: Contours of v_c/T_c on the m_h-m_ξ plane for the one-family model with $N_f = 8$.

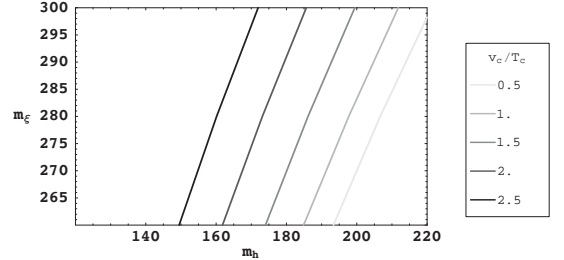FIG. 13: Contours of v_c/T_c on the m_h-m_ξ plane for the non-gauged model with $N_f = 8$.

FIG. 14: Contours of v_c/T_c on the m_h-m_ξ plane for the partially-gauged model with $N_f = 8$.

NG bosons. We note also in this case that ϕ_c/T_c can remain $\mathcal{O}(1)$ even when m_h increases, if at the same time $m_{\xi s}$ increases keeping the relation $m_h < m_{\xi s}$.

VI. SUMMARY AND DISCUSSION

Through the analysis of the finite temperature effective potential at one-loop with the ring-improvement, we have shown that the electroweak phase transition in the SU(2)×U(1) gauged linear sigma model can be strongly first-order with $\phi_c/T_c \simeq \mathcal{O}(1)$, even when the neutral scalar field h, as well as the other scalar fields ξ, is rather heavy: $m_h \simeq 170$GeV, well above the experimental lower bound on the Higgs mass. The explicit symmetry breaking terms, which give rise to the masses of the pseudo NG bosons, reduce the strength of the first-order transition.
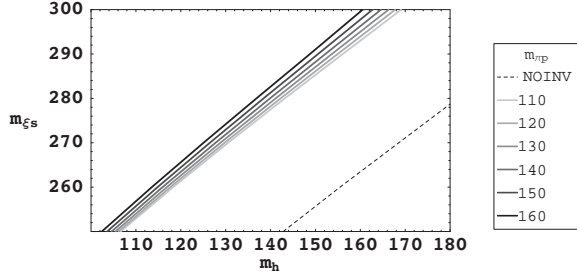
FIG. 15: Contours of $\phi_c/T_c = 1$ on the m_h-$m_{\xi s}$ plane for several values of $m_{\pi p}$ with $m_\eta = 120$ GeV fixed in the non-gauged model with $N_f = 8$. The dashed line shows the contour in the case without the breaking term.
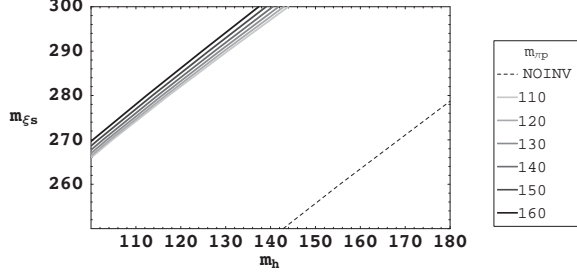

FIG. 16: Contours of $\phi_c/T_c = 1$ on the m_h-$m_{\xi s}$ plane for several values of $m_{\pi p}$ with $m_\eta = 160$ GeV fixed in the non-gauged model with $N_f = 8$.

However, the transition can remain strongly first-order when the masses of pseudo NG bosons are relatively small and in the range $m_\eta, m_{\pi_P} \gtrsim 110$GeV.

It is encouraging to observe the fact that in order to realize the strongly first-order transition with $\phi_c/T_c \simeq \mathcal{O}(1)$, the mass of the neutral scalar field, m_h, can be rather large as long as the masses of the extra scalar fields, m_ξ, are also large and satisfy the relation $m_h < m_\xi$. There does not seem to exist the upper bound on m_h. This does not contradict with the possibility that the restoration of the electroweak symmetry is strongly first-order in the Higgs sector possessing the (approximate) SU(N_f)×SU(N_f) chiral symmetry with a large N_f as in walking technicolor theories.

It would be quite interesting to explore the critical behavior of the restoration of the SU(N_f)×SU(N_f) chiral symmetry with a large N_f in massless QCD- like theories using numerical techniques in lattice field theory. In view of the recent developments of the methods for dynamical simulations in the chiral regime [119–129], this kind of quantitative study from the first principle should become available in quite near future. See [98, 99] for the current status of the numerical study of two-flavor QCD.

It would be also interesting to examine the SU(4)/O(4) chiral symmetry restoration at finite temperature from the point of view of the electroweak phase transition.

This pattern of the chiral symmetry breaking / restoration appears in the minimal walking techicolor theory [81, 82]. In the linear sigma model of this case, the determiant term which breaks the U(1) chiral symmetry is relevant and gives rise to another quartic coupling. It has been argued in [130] that when the derminant term is large, there is a stable IR fixed point which corresponds to a new three-dimensional universality class characterized by the symmetry breaking pattern SU(4)/O(4). (This fixed point does not appear in the ϵ-expansion.) This implies that the finite temperature restoration of the chiral symmetry may be continuous, although it does not exclude a first-order transition for the system that are outside the domain of the stable fixed point. In order to get some insights on the interplay between the low-lying spectrum in the minimal walking techicolor theory and the nature of the critical behavior of the chiral symmetry restoration, a semi-quantitative analysis within the linear sigma model using the finite temperature effective potential may be useful. Results of the numerical studies using lattice field theory has been reported in [131, 132]. See also [133] for a recent attempt to observe the "walking" behavior in the model.

In our analysis of the SU(2)×U(1) gauged linear sigma model, we have used the sphaleron solution to estimate the rate of the baryon number violating interactions. In the case of massless QCD-like theories coupled to the SU(2)×U(1) electroweak interactions, it seems non-trivial to estimate the rate by identifying the counterpart of the sphaleron solution. We leave this question for a future study.

APPENDIX A: EFFECTIVE T-DEPENDENT MASS IN SU(2)×U(1) LINEAR SIGMA MODEL

In this appendix, we show the effective T-dependent masses (mass matrices) for the scalar bosons and the longitudinal components of the gauge bosons in the gauged linear sigma models of the one-family type with $N_f = 8$ and patialy-gauged type with $N_f \geq 3$. In the calculation of the scalar boson self energies and the gauge boson vacuum polarizatoins, we take the IR limit where the Matsubara frequency and the momentum of the external fields go to zero and retain only the leading order terms of $m_i(\phi)/T$. We also show the corresponding degrees of freedom n_i.

1. partialy-gauged type

As the scalar field Φ partialy couples to the gauge fields in this case, it is useful to decompose the field Φ as follows:

$$\Phi = \left(\begin{array}{c|c} \frac{\tilde{h}}{2}\mathbb{1}_2 + \tilde{\Xi} & \frac{1}{\sqrt{2}}\begin{pmatrix} \xi_{13} & \cdots & \xi_{1N_f} \\ \xi_{23} & \cdots & \xi_{2N_f} \end{pmatrix} \\ \hline \frac{1}{\sqrt{2}}\begin{pmatrix} \xi_{13}^* & \xi_{23}^* \\ \vdots & \vdots \\ \xi_{1N_f}^* & \xi_{2N_f}^* \end{pmatrix} & \frac{\check{h}}{\sqrt{2(N_f-2)}}\mathbb{1}_{N_f-2} + \check{\Xi} \end{array}\right)$$

$$+ i\left(\begin{array}{c|c} \frac{\tilde{\eta}}{2}\mathbb{1}_2 + \tilde{\Pi} & \frac{1}{\sqrt{2}}\begin{pmatrix} \pi_{13} & \cdots & \pi_{1N_f} \\ \pi_{23} & \cdots & \pi 2N_f \end{pmatrix} \\ \hline \frac{1}{\sqrt{2}}\begin{pmatrix} \pi_{13}^* & \pi_{23}^* \\ \vdots & \vdots \\ \pi_{1N_f}^* & \pi_{2N_f}^* \end{pmatrix} & \frac{\check{\eta}}{\sqrt{2(N_f-2)}}\mathbb{1}_{N_f-2} + \check{\Pi} \end{array}\right), \quad \text{(A1)}$$

where $\tilde{h}$, $\check{h}$, $\tilde{\eta}$ and $\check{\eta}$ are real fields, $\tilde{\Xi}$ and $\tilde{\Pi}$ are 2×2 traceless Hermitian matrix valued fields, $\check{\Xi}$ and $\check{\Pi}$ are $(N_f - 2) \times (N_f - 2)$ traceless Hermitian matrix valued fields, $\xi_{\alpha\bar{\beta}}$ and $\pi_{\alpha\bar{\beta}}$ ($\alpha = 1, 2; \bar{\beta} = 3, \ldots, N_f$) are complex fields, and $\mathbb{1}_{N_f-2}$ is $(N_f - 2) \times (N_f - 2)$ unit matrix.

The one-loop contributions to the scalar boson self energies are shown in FIG. 5. We use the short-hand notation

$$\Pi^{(S)} = \frac{T^2}{12}[(N_f^2 + 1)\lambda_1 + 2N_f\lambda_2],$$

$$\Pi^{(2)} = \frac{3}{16}g^2 T^2 \ , \ \Pi^{(1)} = \frac{1}{16}g'^2 T^2, \quad \text{(A2)}$$

which are corresponding to the contributions to the scalar boson self energies from the scalar bosons themselves and SU(2), U(1) gauge bosons, respectively.

The effective T-dependent mass matrix for $\tilde{h}$ and $\check{h}$ is

$$\begin{pmatrix} \frac{2}{N_f}m_h^2(\phi) + \frac{N_f-2}{N_f}m_\xi^2(\phi) + \Pi^{(S)} + \Pi^{(1)} + \Pi^{(2)} & \frac{\sqrt{2(N_f-2)}}{N_f}(m_h^2(\phi) - m_\xi^2(\phi)) \\ \frac{\sqrt{2(N_f-2)}}{N_f}(m_h^2(\phi) - m_\xi^2(\phi)) & \frac{N_f-2}{N_f}m_h^2(\phi) + \frac{2}{N_f}m_\xi^2(\phi) + \Pi^{(S)} \end{pmatrix}, \quad \text{(A3)}$$

and each mass eigenvalue has one degree of freedom. The effective T-dependent mass matrix for $\xi_{\alpha\bar{\beta}}$ and $\pi_{\alpha\bar{\beta}}$ is

$$\begin{pmatrix} m_\xi^2(\phi) + \Pi^{(S)} + \frac{1}{2}(\Pi^{(1)} + \Pi^{(2)}) & -\frac{i}{2}(\Pi^{(1)} - \Pi^{(2)}) \\ \frac{i}{2}(\Pi^{(1)} - \Pi^{(2)}) & m_\pi^2(\phi) + \Pi^{(S)} + \frac{1}{2}(\Pi^{(1)} + \Pi^{(2)}) \end{pmatrix}, \quad \text{(A4)}$$

and each mass eigenvalue has $4(N_f - 2)$ degrees of freedom. The other effective T-dependent masses for the scalar bosons are given by

$$\mathcal{M}_{\tilde{h}}^2(\phi, T) = \mathcal{M}_{\tilde{\xi}}^2(\phi, T) = m_\xi^2(\phi) + \Pi^{(S)} + \Pi^{(1)} + \Pi^{(2)},$$

$$\mathcal{M}_{\check{\xi}}^2(\phi, T) = m_\xi^2(\phi) + \Pi^{(S)},$$

$$\mathcal{M}_{\tilde{\eta}}^2(\phi, T) = \mathcal{M}_{\tilde{\pi}}^2(\phi, T) = m_\pi^2(\phi) + \Pi^{(S)} + \Pi^{(1)} + \Pi^{(2)},$$

$$\mathcal{M}_{\check{\pi}}^2(\phi, T) = m_\pi^2(\phi) + \Pi^{(S)}, \quad \text{(A5)}$$

and the corresponding degrees of freedom are

$$n_{\tilde{h}} = n_{\tilde{\eta}} = 1, \ n_{\tilde{\xi}} = n_{\tilde{\pi}} = 3,$$

$$n_{\check{\xi}} = n_{\check{\pi}} = (N_f - 2)^2 - 1. \quad \text{(A6)}$$

For the longitudinal components of the gauge bosons, the one-loop contributions to the vacuum polarizations are shown in FIG. 6. For the W boson,

$$\mathcal{M}_{W_L}^2(\phi, T) = \frac{1}{2N_f}g^2\phi^2 + \frac{N_f + 4}{6}g^2 T^2,$$

$$n_{W_L} = 2. \quad \text{(A7)}$$

The effective T-dependent mass matrix for the Z boson and the photon, in the (A^3, B) basis, is

$$\begin{pmatrix} \frac{1}{2N_f}g^2\phi^2 + \frac{N_f+4}{6}g^2 T^2 & -\frac{1}{2N_f}gg'\phi^2 \\ -\frac{1}{2N_f}gg'\phi^2 & \frac{1}{2N_f}g'^2\phi^2 + \frac{N_f}{6}g'^2 T^2 \end{pmatrix}, \quad \text{(A8)}$$

and each mass eigenvalue has one degree of freedom.

2. one-family type with $N_f = 8$

The one-loop contributions to the scalar boson self energies are shown in FIG. 17. We use the notation (A2) again, for the scalar boson loop, W boson loop and Z boson loop contributions to the scalar boson self-energies.In this model, there is the additional contribution from the gluon loop and we also use the short-hand notation

$$\Pi^{(3)}(\mathbf{8}) = 2g_s^2 T^2, \ \Pi^{(3)}(\mathbf{3}) = \frac{1}{3}g_s^2 T^2, \quad \text{(A9)}$$

which are corresponding to the contributions to the color octet and the color triplet scalar boson self energies from
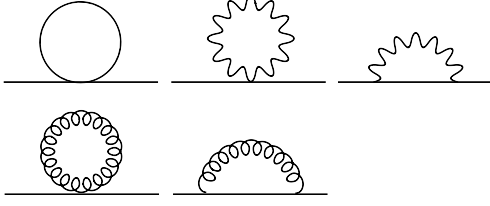
the gluon, respectively.

FIG. 17: The one-loop self energy diagrams of scalar bosons in the one family model.

We show the effective T-dependent masses (and mass matrices) for the scalar bosons on the color irreducible basis.

- Color singlet S and S':

$$\mathcal{M}^2_{h_S}(\phi, T) = m_h^2(\phi) + \Pi^{(S)} + \Pi^{(2)} + \Pi^{(1)}; \ n_{h_S} = 1,$$

$$\mathcal{M}^2_{h_{S'}}(\phi, T) = \mathcal{M}^2_{\xi_S}(\phi, T) = \mathcal{M}^2_{\xi_{S'}}(\phi, T) = m_\xi^2(\phi) + \Pi^{(S)} + \Pi^{(2)} + \Pi^{(1)}; \ n_{h_{S'}} = 1, \ n_{\xi_S} = n_{\xi_{S'}} = 3,$$

$$\mathcal{M}^2_{\eta_S}(\phi, T) = \mathcal{M}^2_{\eta_{S'}}(\phi, T) = m_\eta^2(\phi) + \Pi^{(S)} + \Pi^{(2)} + \Pi^{(1)}; \ n_{\eta_S} = n_{\eta_{S'}} = 1,$$

$$\mathcal{M}^2_{\pi_S}(\phi, T) = \mathcal{M}^2_{\pi_{S'}}(\phi, T) = m_\pi^2(\phi) + \Pi^{(S)} + \Pi^{(2)} + \Pi^{(1)}; \ n_{\pi_S} = n_{\pi_{S'}} = 3. \tag{A10}$$

- Color octet O:

$$\mathcal{M}^2_{h_O}(\phi, T) = \mathcal{M}^2_{\xi_O}(\phi, T) = m_\xi^2(\phi) + \Pi^{(S)} + \Pi^{(3)}(\mathbf{8}) + \Pi^{(2)} + \Pi^{(1)}; \ n_{h_O} = 8, \ n_{\xi_O} = 24,$$

$$\mathcal{M}^2_{\eta_O}(\phi, T) = \mathcal{M}^2_{\pi_O}(\phi, T) = m_\pi^2(\phi) + \Pi^{(S)} + \Pi^{(3)}(\mathbf{8}) + \Pi^{(2)} + \Pi^{(1)}; \ n_{\eta_O} = 8, \ n_{\pi_O} = 24. \tag{A11}$$

- Color triplet T and anti-triplet $\bar{T}$:

  The mass term induce the mixing between T and $\bar{T}$.

  There are four effective T-dependent mass matrices:

$$\begin{pmatrix} \frac{m_\xi^2(\phi)+m_\pi^2(\phi)}{2} + \Pi(\mathbf{3}) & -\frac{24}{9}\Pi^{(1)} & \frac{m_\xi^2(\phi)-m_\pi^2(\phi)}{2} & 0 \\ -\frac{24}{9}\Pi^{(1)} & \frac{m_\xi^2(\phi)+m_\pi^2(\phi)}{2} + \Pi(\mathbf{3}) & 0 & \frac{m_\xi^2(\phi)-m_\pi^2(\phi)}{2} \\ \frac{m_\xi^2(\phi)-m_\pi^2(\phi)}{2} & 0 & \frac{m_\xi^2(\phi)+m_\pi^2(\phi)}{2} + \Pi(\mathbf{3}) & -\frac{24}{9}\Pi^{(1)} \\ 0 & \frac{m_\xi^2(\phi)-m_\pi^2(\phi)}{2} & -\frac{24}{9}\Pi^{(1)} & \frac{m_\xi^2(\phi)+m_\pi^2(\phi)}{2} + \Pi(\mathbf{3}) \end{pmatrix} \tag{A12}$$

  for $(h_T^i, \xi_T^{i,3}, h_{\bar{T}}^i, \xi_{\bar{T}}^{i,3})$, and

$$\begin{pmatrix} \frac{m_\xi^2(\phi)+m_\pi^2(\phi)}{2} + \Pi(\mathbf{3}) & -\frac{24}{9}\Pi^{(1)} & -\frac{m_\xi^2(\phi)-m_\pi^2(\phi)}{2} & 0 \\ -\frac{24}{9}\Pi^{(1)} & \frac{m_\xi^2(\phi)+m_\pi^2(\phi)}{2} + \Pi(\mathbf{3}) & 0 & -\frac{m_\xi^2(\phi)-m_\pi^2(\phi)}{2} \\ -\frac{m_\xi^2(\phi)-m_\pi^2(\phi)}{2} & 0 & \frac{m_\xi^2(\phi)+m_\pi^2(\phi)}{2} + \Pi(\mathbf{3}) & -\frac{24}{9}\Pi^{(1)} \\ 0 & -\frac{m_\xi^2(\phi)-m_\pi^2(\phi)}{2} & -\frac{24}{9}\Pi^{(1)} & \frac{m_\xi^2(\phi)+m_\pi^2(\phi)}{2} + \Pi(\mathbf{3}) \end{pmatrix} \tag{A13}$$

  for $(\eta_T^i, \pi_T^{i,3}, \eta_{\bar{T}}^i, \pi_{\bar{T}}^{i,3})$, and

$$\begin{pmatrix} \frac{m_\xi^2(\phi)+m_\pi^2(\phi)}{2} + \Pi(\mathbf{3}) & \frac{24}{9}\Pi^{(1)} & \frac{m_\xi^2(\phi)-m_\pi^2(\phi)}{2} & 0 \\ \frac{24}{9}\Pi^{(1)} & \frac{m_\xi^2(\phi)+m_\pi^2(\phi)}{2} + \Pi(\mathbf{3}) & 0 & -\frac{m_\xi^2(\phi)-m_\pi^2(\phi)}{2} \\ \frac{m_\xi^2(\phi)-m_\pi^2(\phi)}{2} & 0 & \frac{m_\xi^2(\phi)+m_\pi^2(\phi)}{2} + \Pi(\mathbf{3}) & -\frac{24}{9}\Pi^{(1)} \\ 0 & -\frac{m_\xi^2(\phi)-m_\pi^2(\phi)}{2} & -\frac{24}{9}\Pi^{(1)} & \frac{m_\xi^2(\phi)+m_\pi^2(\phi)}{2} + \Pi(\mathbf{3}) \end{pmatrix} \tag{A14}$$

for $(\xi_T^{i,1}, \pi_T^{i,2}, \xi_{\bar{T}}^{i,1}, \pi_{\bar{T}}^{i,2})$, and

$$
\begin{pmatrix}
\frac{m_\xi^2(\phi)+m_\pi^2(\phi)}{2} + \Pi(\mathbf{3}) & -\frac{24}{9}\Pi^{(1)} & \frac{m_\xi^2(\phi)-m_\pi^2(\phi)}{2} & 0 \\
-\frac{24}{9}\Pi^{(1)} & \frac{m_\xi^2(\phi)+m_\pi^2(\phi)}{2} + \Pi(\mathbf{3}) & 0 & -\frac{m_\xi^2(\phi)-m_\pi^2(\phi)}{2} \\
\frac{m_\xi^2(\phi)-m_\pi^2(\phi)}{2} & 0 & \frac{m_\xi^2(\phi)+m_\pi^2(\phi)}{2} + \Pi(\mathbf{3}) & -\frac{24}{9}\Pi^{(1)} \\
0 & -\frac{m_\xi^2(\phi)-m_\pi^2(\phi)}{2} & \frac{24}{9}\Pi^{(1)} & \frac{m_\xi^2(\phi)+m_\pi^2(\phi)}{2} + \Pi(\mathbf{3})
\end{pmatrix}
\tag{A15}
$$

for $(\xi_T^{i,2}, \pi_T^{i,1}, \xi_{\bar{T}}^{i,2}, \pi_{\bar{T}}^{i,1})$, where

$$
\Pi(\mathbf{3}) = \Pi^S + \Pi^{(3)}(\mathbf{3}) + \Pi^{(2)} + \frac{25}{9}\Pi^{(1)}.
\tag{A16}
$$

These four matrices have same eigenvalues. After the diagonalization of these mass matrices, we obtain the effective T-dependent masses for the color (anti-) triplet scalar bosons. The eigenvalues are

$$
m_\xi^2(\phi) + \Pi(\mathbf{3}) \pm \frac{8}{3}\Pi^{(1)}, \; m_\pi^2(\phi) + \Pi(\mathbf{3}) \pm \frac{8}{3}\Pi^{(1)},
\tag{A17}
$$

and each mass eigenvalue has 12 degrees of freedom.

For the longitudinal components of the gauge bosons, the one-loop contributions to the vacuum polarizations are shown in FIG. 6. For the W boson,

$$
\mathcal{M}_{W_L}^2(\phi, T) = \frac{1}{4}g^2\phi^2 + 6g^2T^2, \; n_{W_L} = 2.
\tag{A18}
$$

The effective T-dependent mass matrix for the Z boson and the photon, in the (A^3, B) basis, is

$$
\begin{pmatrix}
\frac{1}{4}g^2\phi^2 + 6g^2T^2 & -\frac{1}{4}gg'\phi^2 \\
-\frac{1}{4}gg'\phi^2 & \frac{1}{4}g'^2\phi^2 + \frac{80}{9}g'^2T^2
\end{pmatrix},
\tag{A19}
$$

and each mass eigenvalue has one degree of freedom.

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